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**COMPARISON OF FATIGUE LIFE ESTIMATION USING  
EQUIVALENT LINEARIZATION AND TIME DOMAIN  
SIMULATION METHODS**

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## ABSTRACT

### Comparison of Fatigue Life Estimation Using Equivalent Linearization and Time Domain Simulation Methods

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The Monte Carlo simulation method in conjunction with the finite element large deflection modal formulation are used to estimate fatigue life of aircraft panels subjected to stationary Gaussian band-limited white-noise excitations. Ten loading cases varying from 106 dB to 160 dB OASPL with bandwidth 1024 Hz are considered. For each load case, response statistics are obtained from an ensemble of 10 response time histories. The finite element nonlinear modal procedure yields time histories, probability density functions (PDF), power spectral densities and higher statistical moments of the maximum deflection and stress/strain. The method of moments of PSD with Dirlik's approach is employed to estimate the panel fatigue life.

## FOREWORD

This technical report summarizes the research results on fatigue life estimation of aircraft panel subjected to acoustic random excitations using the time domain numerical simulation and the finite element nonlinear modal formulation methods. The research was sponsored by a NASA Langley Research Center grant NAG-1-2294, entitled “Comparison of Fatigue Life Estimation Using Equivalent Linearization and Time Domain Simulation Methods.” Technical monitor was Dr. Stephen A. Rizzi, Structural Acoustics Branch, NASA Langley Research Center. The authors generated the response statistics and estimated the fatigue life of an aircraft panel using the simulation and the nonlinear finite element modal methods. Dr. Stephen A. Rizzi obtained fatigue life estimation using the equivalent linearization method and made the comparison of the fatigue life estimations.

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## 1. INTRODUCTION

For sonic fatigue design and analysis of aircraft panels subjected to a random excitation environment, the link between the dynamic structural response and the fatigue life estimation of the component seems to be far apart. The task of this project is to perform and compare the fatigue life estimations using the equivalent linearization and the time domain Monte Carlo simulation approaches.

### 1.1 Equivalent Linearization

The technique of statistical linearization is based on the concept of replacing the nonlinear system with a related linear system such that the difference between the two systems is minimized. Basically the method is the statistical extension of well-known Krylov-Bogoliubov equivalent linearization technique for deterministic vibration problems. The extension of this technique to problems of random excitation was independently developed by Booton [1] and Caughey [2]. Kazakov [3] has generalized the equivalent linearization method to the case of multidimensional inertial nonlinearities. Atalik and Utku [4] have provided a more direct and simplified version of Kazakov's formulation as applied to Multi-Degree-of-Freedom (MDOF) nonlinear systems that may be nonlinear in inertial, velocity and restoring forces. A comprehensive account of the equivalent linearization method is referred to the book by Roberts and Spanos [5]. The equivalent linearization assumes that the response is Gaussian distribution for the reduction of higher even order statistical moments to the second-order moments.

Applications of the finite element analysis (FEA) and the equivalent linearization to sonic fatigue of aircraft panels have been presented by Chiang [6], and Locke [7]. The equivalent linearization approach has been implemented in FEA codes by Rizzi et al.,

Structural Acoustics Branch at NASA Langley Research Center [8-10]. The equivalent linear stiffness matrix through the force error or the potential energy error minimization is obtained with an iterative scheme of equivalent linearization [9,10].

## **1.2 Simulation Method**

Time domain numerical simulation or the Monte Carlo method estimates the response statistics of randomly excited nonlinear structural systems [11-13]. The method mainly consists of generating a large number of sample excitations, calculating the corresponding response samples, and processing the response samples to obtain the desired response statistics [12]. Obviously this approach can be used for estimating the response statistics of both stationary and non-stationary systems. The main drawback of this method is the computation time and cost.

Vaicaitis et al. have employed the Galerkin's method (Partial Differential Equation, PDE, and modal approach) and the Monte Carlo simulation for the prediction of isotropic [13-15] and composite [16,17] panels subjected to acoustic and thermal loads. The use of PDE/Galerkin method, however, limits its applicability to simple rectangular panel planform and simple boundary conditions.

Applications of the FEA and the time domain numerical simulation to aircraft panels were presented by Robinson [18], and Green and Killey [19]. Their finite element models are formulated in the structural node degree-of-freedom (DOF). This approach turned out to be extremely computationally costly because of: (i) the large number of DOF of the nonlinear system, (ii) the nonlinear stiffness matrices have to be assembled and updated from the element nonlinear stiffness matrices at each time step, and (iii) the time step of integration should be extremely small. The disadvantage of costly long

computation time can be remedied by transformation the finite element equations from the structural node DOF to the truncated modal amplitudes [20,21]. The accuracy of the nonlinear stiffness matrix formulation in modal amplitudes has been validated for nonlinear free vibration [20] and nonlinear random vibration [21] of isotropic and composite panels. The time domain numerical simulation to aircraft panels [21] yields time histories, probability distribution functions (PDF), power spectral densities (PSD) and higher statistical moments of the maximum deflection and strain/stress. The moments of PSD of maximum strain/stress are used for fatigue life estimation.

## 2. FATIGUE LIFE ESTIMATION

The estimate of fatigue life is based on the relevant spectral moments [22,23] which are computed from the one-sided PSD,  $G(f)$ , in units of Hertz as

$$m_n = \int_0^\infty f^n G(f) df \quad (1)$$

Rice [24] has developed the important relationships for the number of upward zeros per second,  $E[0]$ , and peaks per second,  $E[P]$ , in a random signal expressed solely in terms of spectral moments  $m_n$  as

$$\begin{aligned} E[0] &= (m_2/m_0)^{1/2} \\ E[P] &= (m_4/m_2)^{1/2} \end{aligned} \quad (2)$$

The irregular factor  $\gamma$ , a very useful term for characterizing the behavior of structural response, was defined as

$$\gamma = \frac{E[0]}{E[P]} = \frac{m_2}{(m_0 m_4)^{1/2}} \quad (3)$$

which varies between 0 (white noise containing a broad range of frequencies) and 1 (a classical narrow band signal containing only one predominant frequency). The fatigue analysis procedures presented by Bishop et al. [22,23] with Dirlik's [25] expression for the PDF of rainflow ranges is used to estimate fatigue life. Dirlik expression for rainflow ranges is

$$p(S) = \frac{\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}}}{2\sqrt{m_0}} \quad (4)$$

where  $p(S)$  is the probability density function of rainflow ranges of  $S$ , and

$$\alpha = \frac{m}{\sqrt{m_0 m_4}} \quad (5)$$

$$x_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_4}} \quad (6)$$

$$D_1 = \frac{2(x_m - \alpha^2)}{1 + \alpha^2} \quad (7)$$

$$D_2 = \frac{(1 - \alpha - D_1 + D_1^2)}{1 - R} \quad (8)$$

$$D_3 = 1 - D_1 - D_2 \quad (9)$$

$$Q = \frac{1.25(\alpha - D_3 - D_2 R)}{D_1} \quad (10)$$

$$R = \frac{\alpha - x_m - D_1^2}{1 - \alpha - D_1 + D_1^2} \quad (11)$$

$$Z = \frac{S}{2\sqrt{m_0}} \quad (12)$$

and the stress range  $S$



$$S = 2\sqrt{m_0}Z \quad (13)$$

For random loads of variable amplitude, the  $S$ - $N$  curves and the Dirlik's empirical relation are combined by means of the *Palmgren-Miner* linear damage accumulation theory, to predict fatigue failure. The Palmgren-Miner hypothesis is that the fatigue damage incurred at a given load level is proportional to the sum of the number of cycles applied at that stress level divided by the total number of cycles required to cause failure at the same level. When the fatigue loading involves many levels of stress amplitude, the total damage is a sum of the different cycle ratios and failure occurs when the cycle ratio sum equals one

$$D = \sum_{k=1}^p \left( \frac{n_i}{N_i} \right) = 1.0 \quad (14)$$

Combining the Dirlik's approach with Miner's failure criteria the total damage is given by

$$E[D] = E[P] \frac{T}{K} \int_0^\infty S^\beta p(S) dS = E[P] \cdot \frac{T}{K} \sum_{S < S_k} S^\beta p(S) \quad (15)$$

where  $E[P]$  is the expected number of peaks defined as  $E[P] = \sqrt{m_4/m_2}$ ,  $T$  the duration of the measurement and  $S_k$  is the maximum design value of the ultimate stresses that for aluminum structures are in the neighborhood of 45,000 to 55,000 psi in areas which are fatigue critical.

## 2.1 Simulated Input White-Noise

The band-limited white-noise is simulated by a Fortran code that generates a random pressure using complex numbers with independent random phase angles uniformly distributed between 0 and  $2\pi$ . A total of 10 load cases have been considered, shown in

Table 1. The broad range of load cases varies from the low level of 106 dB overall sound pressure level (OASPL) to the high level of 160 dB OASPL. A typical time history corresponding corresponding to load case 7 is shown in Figure 1. The corresponding PDF is shown with the Gaussian distribution. The PSD gives a spectrum level of  $5.3848 \times 10^{-6} \text{ (psi)}^2/\text{Hz}$  over 1024 Hz bandwidth.

Table 1. 10 Load Cases

Case No.	ref: Case 0	psi	OASPL (dB)	SPL (dB)	psi <sup>2</sup> /Hz
0	1	0.0005801	106.0206	75.9176	3.2866E-10
1	2	0.0011603	112.0412	81.9382	1.3147E-09
2	4	0.0023205	118.0618	87.9588	5.2586E-09
3	8	0.004641	124.0824	93.9794	2.1034E-08
4	16	0.0092821	130.1030	100.0000	8.4138E-08
5	32	0.0185642	136.1236	106.0206	3.3655E-07
6	64	0.0371284	142.1442	112.0412	1.3462E-06
7	128	0.0742567	148.1648	118.0618	5.3848E-06
8	256	0.1485134	154.1854	124.0824	2.1539E-05
9	512	0.2970268	160.2060	130.1030	8.6157E-05

## 2.2 Panel Modeling and Response Processing

A 14x10x0.04 in. (35.6x25.4x0.1 cm) simply supported aluminum plate with material properties of  $E=10.587 \text{ Msi}$  (73 GPa),  $\nu=0.3$ , and  $\rho=2.588 \times 10^{-4} \text{ lbf-sec}^2/\text{in.}^4$  (2763 kg/m<sup>3</sup>) is studied. The inplane boundary is assumed to be immovable with  $u(0,y)=u(a,y)=v(x,0)=v(x,b)=0$ . The panel is modeled with a 14x10 mesh or 140 Bogner-Fox-Schmidt (BFS) C<sup>1</sup> conforming rectangular elements in a quarter plate model. The two lower modes (1,1) and (3,1) are included in the response analyses for comparison of fatigue life estimation using the equivalent linearization technique. A proportional damping ratio of  $\xi_r \omega_r = \xi_s \omega_s$  with  $\xi_1=0.02$  is used. An explicit fourth-order Runge-Kutta numerical integration scheme is employed. The time step of integration was selected

based on by reducing the time step by one half until identical displacement response was obtained for two different consecutive time steps.

The panel is initially at rest, an initial transient response is therefore induced before the response becomes fully developed. The transient response must be eliminated to ensure that the accurate response statistics is recovered. For each input loading time history, a total response of one second was taken out of two seconds constituting the total run. The first and last 0.5 sec response was discarded. Response statistics were generated from an ensemble of 10 time histories at each load case.

### 2.3 Results

Response statistics for load case 1 (112 dB OASPL) and case 7 (148 dB OASPL) are given in the following. The results include time histories, probability distribution functions (PDF), power spectral densities (PSD) (Figures 2-5), fatigue life estimates (Table 2) and higher statistical moments (Table 3) of the maximum deflection and strain/stress. Results for all 10 cases have been forwarded to Dr. Stephen Rizzi at Langley Research Center.

Table 2. Fatigue Life Estimates Using Dirlik's Approach

Fatigue Life in hours	
Case 1	$3.59 \times 10^6$ or $\infty$
Case 7	55.8

**Table 3**  
RMS and Moments of the  $W_{\max}$  and  $(\sigma_1)_{\max}$   
for the 14x10x0.04 in. Isotropic Plate

	RMS	Mean	Variance	Skewness	Kurtosis
	$\mu\text{.m.}$	$\mu\text{.m.}$	$\mu\text{.m}^2/\text{m}^2$	$\text{m}^3/\text{m}^3$	$\text{m}^4/\text{m}^4$
	$W_{\max}$				
Case 1	0.512	0.3068	0.00262	0.0131	0.176
Case 7	1205	2.583	1.451	-0.00628	-0.621
	Stress				
	M.Pa	M.Pa	G.Pa <sup>2</sup> /Pa <sup>2</sup>	Pa <sup>3</sup> /Pa <sup>3</sup>	Pa <sup>4</sup> /Pa <sup>4</sup>
Case 1	0.376	0.00192	141.85	0.00589	0.156
Case 7	8.990	-0.173	8079.2	-0.0477	-0.510

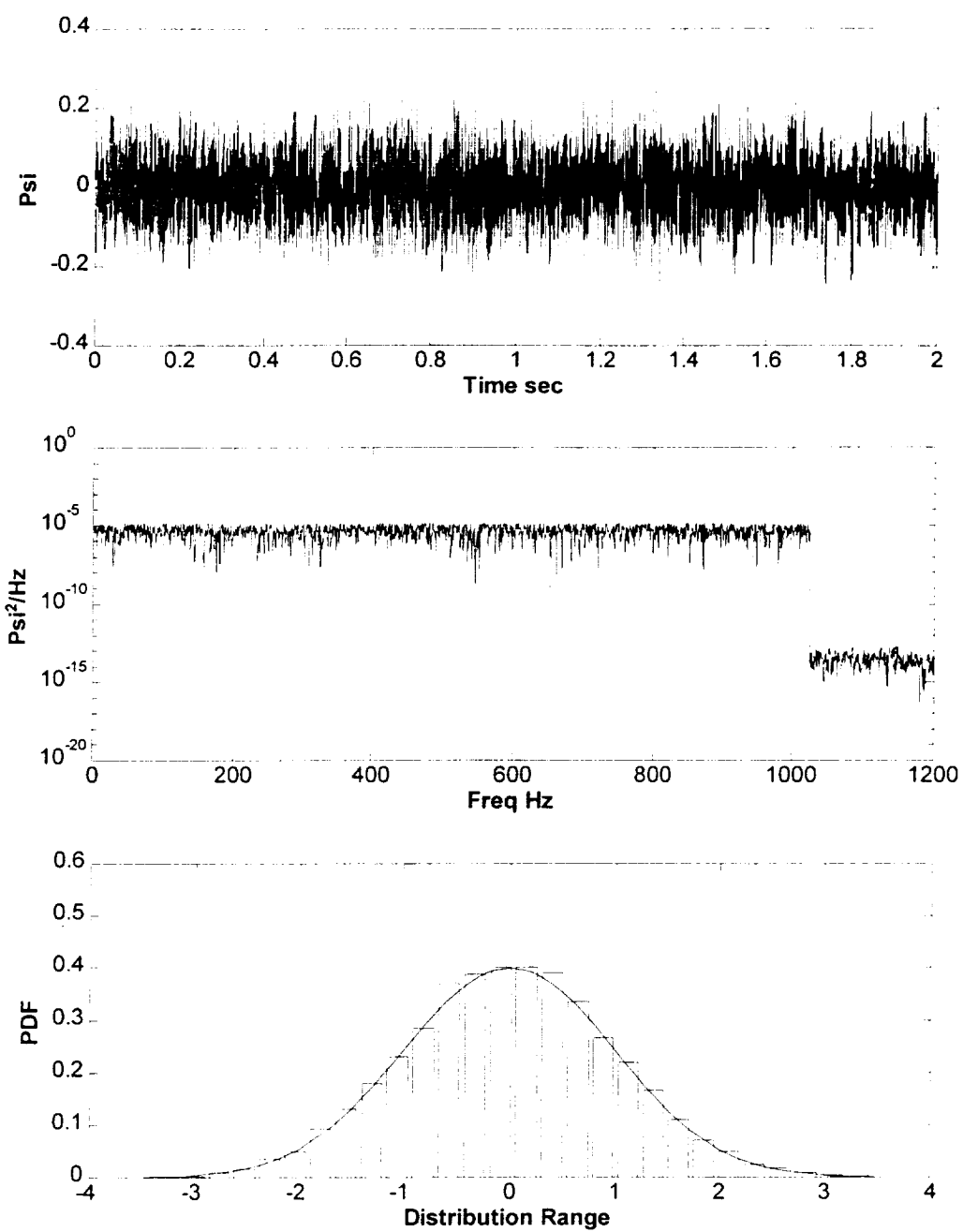


Figure 1. Load for Case 7

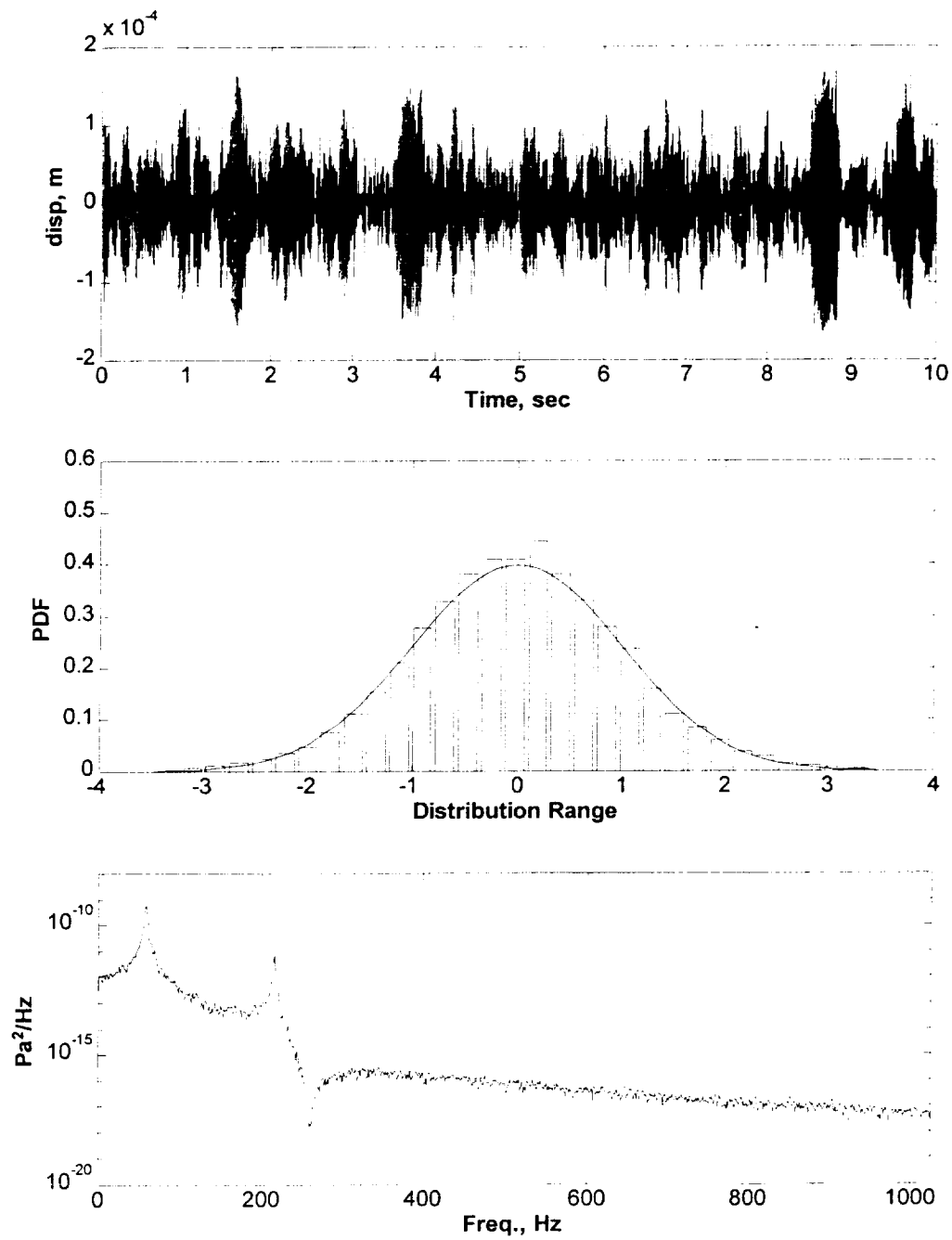


Figure 2. Time History, PDF and PSD of Displacement for Case 1

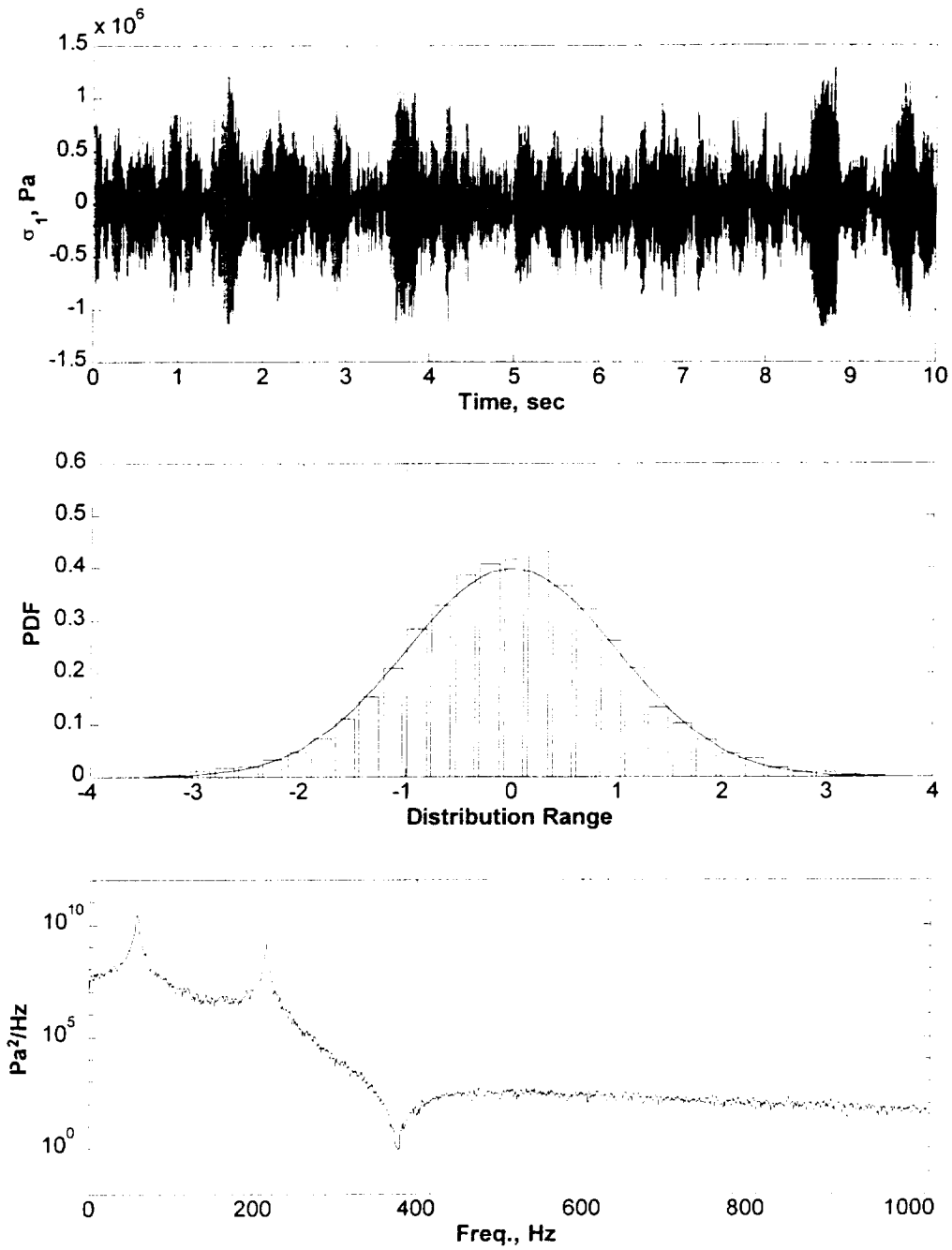


Figure 3. Time History, PDF and PSD of Principal Stress for Case 1

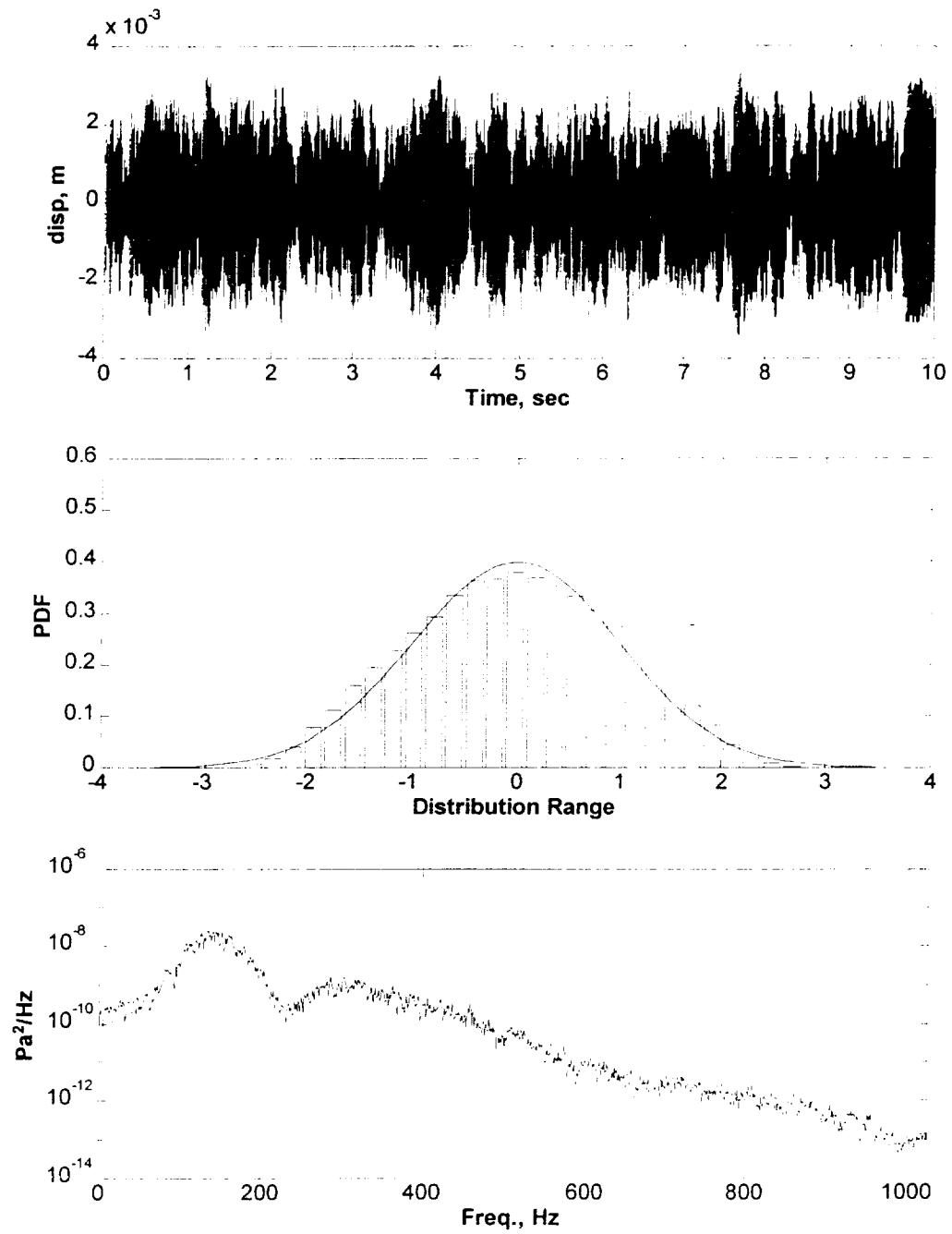


Figure 4. Time History, PDF and PSD of Displacement for Case 7



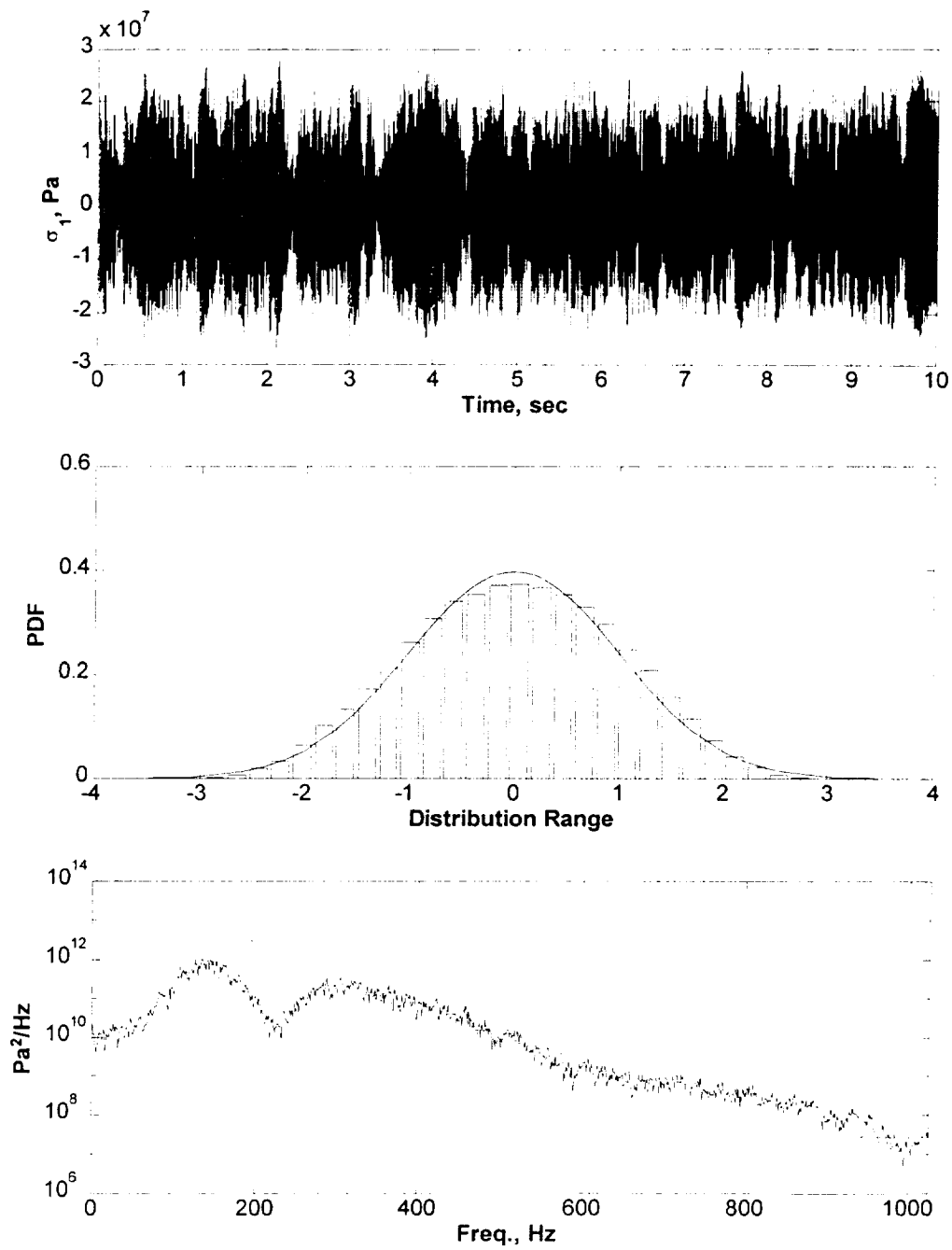


Figure 5. Time History, PDF and PSD of Principal Stress for Case 7

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